AN ENTROPY APPROACH FOR THE OPTIMIZATION OF RIVER CROSS-SECTIONAL SPACING

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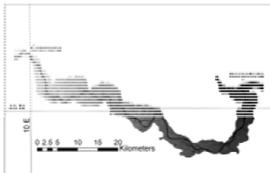
Introduction

- **Accurate definition of rivers geometry is a relevant task in preparation of one-dimensional hydraulic models. In particular, the selection of river cross sections spacing is key for accurately describing the river hydraulics.**
- **Optimization of cross sectional spacing allows the modeler to lighten the hydraulic model of the river and to reduce the computational time.**
- **In this work an entropy approach, that allows evaluating the optimum number of river cross sections and their location on the river geometry, is presented.**
- **The problem is posed as a MultiObjective Optimization Problem (MOOP), minimizing total correlation (as a measure of redundancy) and maximizing joint entropy (as a measure of information content).**
- **The MOOP is solved using the non-sorted genetic algorithm NSGA-II and a Pareto front of optimal sets of cross sectional locations is obtained. The Pareto front is then analyzed and the most appropriate solution is chosen.**

Case study

The Po river is the main Italian river with a length of approximately 650 km and a drainage area of about 71,000 km2. The area considered in this work is a 98 km reach between Cremona and Borgoforte, Figure (1). The Po River Basin Authority provided an high quality 2 m digital terrain model (DTM); as boundary condition was used the 1-in-200 years flood (Brandimarte and Di Baldassarre, 2011).



Po river between Cremona and Borgoforte and LIDAR topography (grey scale).

Literature guideline

In case of backwater at the end of the river reach, the best spacing distance among cross sections can be calculated as (Castellarin et al., 2009; Samuels, 1990) :

$$\Delta x < 0.2\frac{(1-F^2)D}{s}$$

Where F is the Froud's number, D is the bankfull depth of flow, s is the slope of the channel.

Po River	*D (m)*	*s*	*F*	*Δx (km)*	*n. sections*
Average	14.00	14*E-5	0.12	20	4
Max	21.82	14*E-5	0.18	30	3

The entropy method

A multiobject function has been considered (MOOP). The set of the optimal cross sections have to fulfill (Alfonso, 2010a,b):

$$Min(C) = Min\{C(X_1, X_2,..., X_N)\}$$
$$Max(JH) = Max\{H(X_1, X_2,..., X_N)\}$$

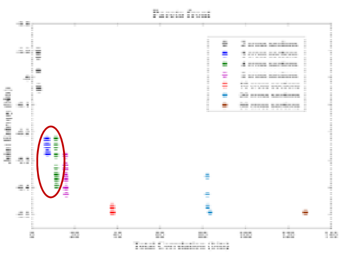
Where C is the total correlation:

$$C(X_1, X_2,...X_N) = \sum_{i=1}^{N} H(X_i) - H(X_1, X_2,..X_N)$$

JH is the joint entropy, and p is the joint probability of a particular combination of the water stage in N different cross sections:

$$H(X_1,..., X_N) = -\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_N=1}^{N_N} p_{i_1,...,i_N}\log_2(p_{i_1,...,i_N})$$

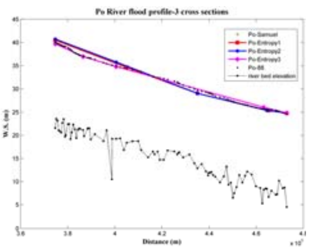
Pareto front describes the optimal set of solutions discriminated by the NSGA-II (Deb et al., 2002).



Between 4 and 5 sections, the difference in term of redundancy is 4.4290 bits, while the gain in term of joint entropy is only 0.0306 bits.

It is not worth in using more cross sections than 4, because there is no information gain and an high addition of redundancy.

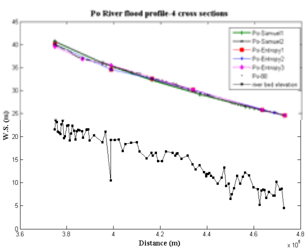
3 cross sections



Model	*MAE*
Po-Samuels1	0.315
Po-Entropy1	0.315
Po-Entropy2	0.492
Po-Entropy3	0.270

There is a perfect match between Samuels' solution and Po-Entropy1. Besides, the entropy method provides different combinations for the same amount of sections, among them Po-Entropy3 has the lowest MAE.

4 cross sections



Model	*MAE*
Po-Samuels1/2	0.159
Po-Entropy1	0.304
Po-Entropy2	0.323
Po-Entropy3	0.332

Two set of cross sections give the lowest MAE. Flood profiles match , with Po-88, built with the whole set of sections, better than the model with 3 sections.

Conclusions

- **Results obtained using the entropy method match with those provided by Samuels' guideline.**
- **In using the entropy method, for the same number of cross sections, many configurations are possible. The modeler can choose the one with the lowest MAE.**
- **The information difference between 3 and 4 sections, noticeable in the Pareto front, afflicts the flood profiles, giving a less precise curve.**
- **Thus, the optimal number of cross sections is 4, using the entropy method.**

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